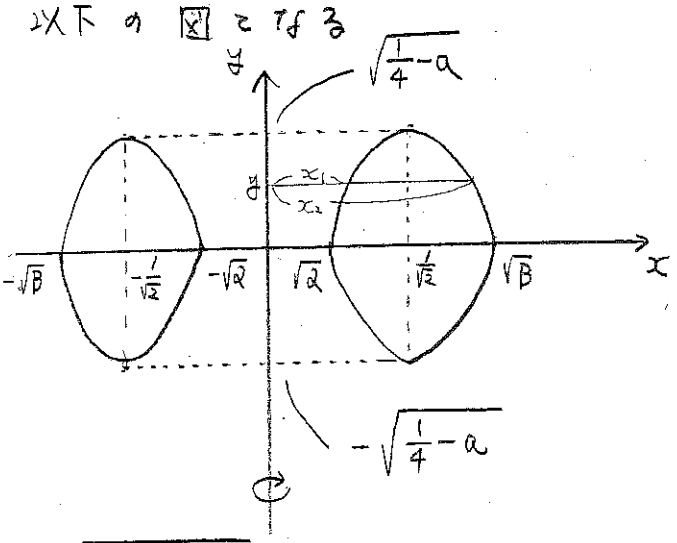
よていは考えている領域は ①, ② より
以下の図となる



① $y^2 \leqq x^2(1-x^2)-a$

$y^2 \leqq -x^4+x^2-a \quad (0<a<\frac{1}{4})$

$-\sqrt{-x^4+x^2-a} \leqq y \leqq \sqrt{-x^4+x^2-a}$ ——— ①

ここで

$y=\sqrt{-x^4+x^2-a}$

$=\sqrt{-(x^2-\frac{1}{2})^2+\frac{1}{4}-a}$ ——— ①'

①' のグラフが存在するための a のハニイは

$-x^4+x^2-a \geqq 0$

$x^4-x^2+a \leqq 0$

$\frac{1-\sqrt{1-4a}}{2} \leqq x^2 \leqq \frac{1+\sqrt{1-4a}}{2}$

α ,, β とおく $(\alpha<\beta)$

$\alpha \leqq x^2 \leqq \beta$

$\begin{cases} \alpha \leqq x^2 \leftrightarrow (x^2-\alpha) \geqq 0 \\ x^2 \leqq \beta \leftrightarrow (x^2-\beta) \leqq 0 \end{cases}$

$\begin{cases} x \leqq -\sqrt{\alpha},\ \sqrt{\alpha} \leqq x \\ -\sqrt{\beta} \leqq x \leqq \sqrt{\beta} \end{cases}$ ——— ②

$y=\sqrt{-x^4+x^2-a}$

$y^2=-x^4+x^2-a$

$x^4-x^2+a+y^2=0$

$x^2=\frac{1\pm\sqrt{1-4a-4y^2}}{2}=x_1^2,\ x_2^2$ とおく $(x_1<x_2)$

$\frac{V}{2}=\int_0^{\sqrt{\frac{1}{4}-a}} \pi x_2^2\,dy-\int_0^{\sqrt{\frac{1}{4}-a}} \pi x_1^2\,dy$

$=\int \pi(x_2^2-x_1^2)\,dy$

$=\pi\int\sqrt{1-4a-4y^2}\,dy$

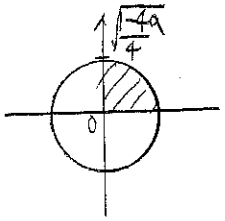
$=2\pi\int\sqrt{\frac{1-4a}{4}-y^2}\,dy$

$=2\pi\cdot\pi\left(\sqrt{\frac{1-4a}{4}}\right)^2\cdot\frac{1}{4}$

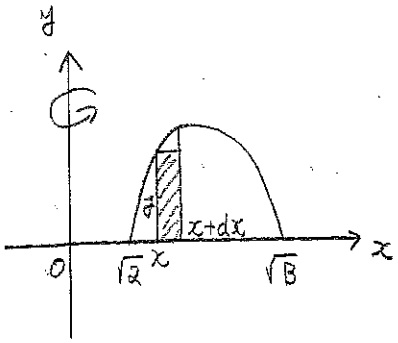
$=\frac{\pi^2}{2}\cdot\frac{1-4a}{4}$

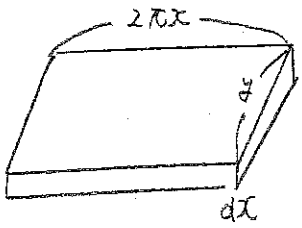
$=\frac{\pi^2}{8}(1-4a)$

$\therefore V=\frac{\pi^2}{4}(1-4a)$ //



(別)





$$\frac{V}{2} = \int_{\sqrt{2}}^{\sqrt{\beta}} 2\pi x y \, dx$$

$$= \int_{\sqrt{2}}^{\sqrt{\beta}} 2\pi x \sqrt{-(x^2-\frac{1}{2})^2+\frac{1}{4}-a} \, dx$$ ——— ☆

$x^2-\frac{1}{2}=t$ とおく

$2x\,dx = dt$

x	$\sqrt{2} \longrightarrow \sqrt{\beta}$
t	$2-\frac{1}{2} \longrightarrow \beta-\frac{1}{2}$

$$2-\frac{1}{2} = \frac{1-\sqrt{1-4a}}{2} - \frac{1}{2} = \frac{-\sqrt{1-4a}}{2}$$

$$\beta-\frac{1}{2} = \frac{1+\sqrt{1-4a}}{2} - \frac{1}{2} = \frac{\sqrt{1-4a}}{2}$$

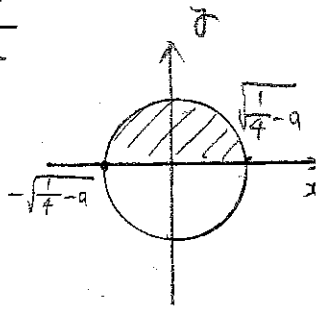
☆より

$$\int_{\frac{-\sqrt{1-4a}}{2}}^{\frac{\sqrt{1-4a}}{2}} \pi \sqrt{-t^2+\frac{1}{4}-a} \, dt$$

$$= \pi \int_{-\sqrt{\frac{1}{4}-a}}^{\sqrt{\frac{1}{4}-a}} \sqrt{\frac{1}{4}-a-x^2} \, dx$$

$$= \pi \cdot \pi\left(\sqrt{\frac{1}{4}-a}\right)^2 \cdot \frac{1}{2}$$

$$= \left(\frac{1}{4}-a\right)\frac{1}{2}\pi^2$$



$$\therefore V = \pi^2\left(\frac{1}{4}-a\right)$$

$$= \frac{\pi^2}{4}(1-4a)$$